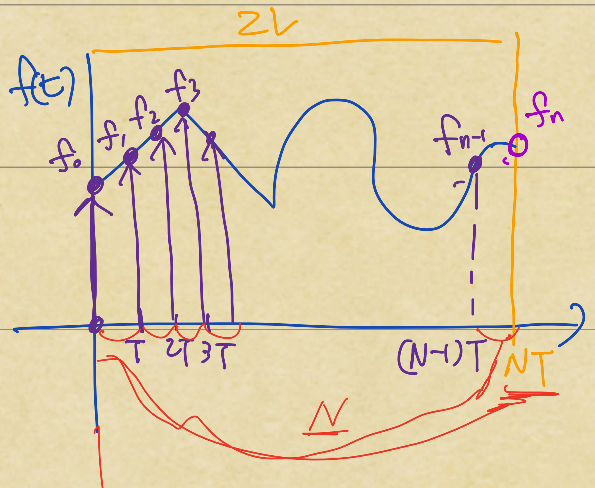
- Fourier Series

$$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{i\omega t}, \quad C_k = \frac{1}{2L} \langle\langle f(x) . e^{i\omega t} \rangle\rangle = \frac{1}{2L}\int_{-\infty}^{\infty} f(x) \cdot e^{-i\omega t} dt$$

- Fourier transform $\hat{f}(\omega) = F(f(t)) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i\omega t} dt$

- Discrete Fourier transform (DFT)



$\underline{N \text{ samples}}$ $f_0 \; f_1 \ldots f_{n-1}$

T = sampling interval

$f_s = \frac{1}{T}$ = sampling rate (Hz)

\# samples / sec

$\underbrace{NT} = 2L$

$$F(f(t)) \Rightarrow F(\underbrace{\{f[N]\}}_{f_0 \cdots f_{n-1}}) = \int_0^{NT} f(t) \cdot e^{-i\omega t} dt$$

$$= \int_0^{T} f_0 \delta(t-0) \cdot e^{-i\omega t} dt + \int_T^{2T} f_1 \delta(t-T) \cdot e^{-i\omega t} dt$$

$$+ \int_{2T}^{3T} f_2 \delta(t-2T) \cdot e^{-i\omega t} dt \cdots + \int_{(N-1)T}^{NT} f_{n-1} \delta(t-(N-1)T) \cdot e^{-i\omega t} dt$$

$$= f_0 \cdot e^{-i\omega 0} + f_1 \cdot e^{-i\omega T} + f_2 \cdot e^{-i\omega 2T} \cdots + f_{n-1} e^{-i\omega (N-1)T}$$

$$\hat{f}(\omega) = \sum_{j=0}^{n-1} f_j \cdot e^{-i\boxed{\omega} jT}$$

$\omega = \frac{2k\pi}{2L} = \frac{2k\pi}{NT}$

$k=0$, DC term $\sum_{0}^{n-1} f_j$

$k=1$, Fundamental freq

$k=2, 3, \vdots$ } harmonics

$$= \sum_{j=0}^{n-1} f_j \cdot e^{-i\frac{2k\pi}{N} j \cancel{T}} = \sum_{j=0}^{n-1} f_j\, \omega_N^{kj}$$

$$\boxed{\omega_N = e^{-\frac{i2\pi}{N}}}$$

DFT matrix

$$
\begin{array}{ccc}
0 & 0 \cdot f_s & k=0 \\
\frac{f_s}{N} & \frac{1}{NT} & k=1 \\
\frac{2f_s}{N} & \frac{2}{NT} & k=2 \\
\frac{3f_s}{N} & \frac{3}{NT} & k=3 \\
\vdots & & \\
\frac{f_s}{2} & & \\
 & & k=n-1 \\
\boxed{f_s} & \frac{N}{NT} & k=n
\end{array}
\quad
\begin{bmatrix} \hat{f}_0 \\ \hat{f}_1 \\ \hat{f}_2 \\ \vdots \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
1 & 1 & 1 & \cdots & \\
\omega_N^0 & \omega_N^1 & \omega_N^2 & \cdots & \omega_N^{N-1} \\
\omega_N^0 & \omega_N^2 & \omega_N^4 & \cdots & \omega_N^{2(N-1)} \\
\vdots & & & & \\
\omega_N^0 & \omega_N^{n-1} & \omega_N^{(n-1)2} & \cdots & \omega_N^{(n-1)^2}
\end{bmatrix}
\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_{n-1} \end{bmatrix}
\begin{array}{l} j=0 \\ j=1 \\ \\ \\ j=n-1 \end{array}
$$

$N \times N$ what

N samples

Bogus!!

$\hat{f}_k$ = complex number < manitude this $f(t)$ / phase

Tn (what

$a + ib$

$r + \theta i$

Similar to $e^{-i\omega t}$ $\quad \omega = \frac{k\pi}{L}$

$\boxed{\text{FFT}}$ is a fast implementation of DFT

$O(N \log N)$

$$\begin{bmatrix} \ddots & & 0 \\ & \ddots & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \end{bmatrix} \Big\} N \quad \Big\} \log_2 N$$

$$\hat{f} = \underline{\text{fft}}\,(\underbrace{\overline{\text{signal}}}_{N})$$